PROBABILITY
AND
MATHEMATICAL STATISTICS
Vol. 20, Fasc. 2 (2000), pp. 383–390

ON HARMONIC MEASURE FOR LÉVY PROCESSES

BY

PAWEŁ SZTONYK* (WROCŁAW)

Abstract. Let $\{X_t\}$ be a Lévy process in $\mathbf{R}^d$, $d \geqslant 2$, with infinite Lévy measure. If $\{X_t\}$ has no Gaussian component, then the process does not hit the boundary of Lipschitz domain $S \subset \mathbf{R}^d$ at the first exit time of S under mild conditions on $\{X_t\}$. The conditions are met, e.g., if $\{X_t\}$ is rotation invariant.

1991 MS Classification: Primary 60J30, 60G17; Secondary 60G40, 60J25.

Key words and phrases: Lévy processes, harmonic measure, Lipschitz domains, Lévy measure.

1. Introduction. The paper is concerned with the properties of the harmonic measure of Lévy process $\{X_t\}_{t \geqslant 0}$ in $\mathbf{R}^d$. Recall that for an open set $S \subset \mathbf{R}^d$ the harmonic measure ω_S^x corresponding to $\{X_t\}_{t \geqslant 0}$ is defined as $\omega_S^x(\cdot) = P^x(X_{\tau_S} \in \cdot)$, where $x \in \mathbf{R}^d$ and $\tau_S = \inf\{t > 0\colon\ X_t \notin S\}$. For the Brownian motion in $\mathbf{R}^d$ the continuity of paths yields $\omega_S^x(\partial S) = 1$ for every domain S and all $x \in S$. However, in general, $\{X_t\}_{t \geqslant 0}$ has discontinuous paths and, depending on the process and the domain, it may happen that with probability one the process does not hit the boundary of S when first leaving it. For example, for the rotation invariant stable Lévy process in $\mathbf{R}^d$ we have $\omega_S^x(\partial S) = 0$, provided S is a bounded domain with the outer cone property (see [2], Lemma 6). If $\omega_S^x(\partial S) = 0$ for $x \in S$, then the harmonic measure may be completely described by means of the Green function and Lévy measure via a formula of Ikeda and Watanabe [3]. This motivates the search for general conditions on the Lévy process and the domain guaranteeing the *discontinuous exit property*: $\omega_S^x(\partial S) = 0,\ x \in S$.

In the one-dimensional case this exit problem was completely solved in [5] in the case where S is a half-line. The problem becomes much more difficult to handle in $\mathbf{R}^d$ for $d \geqslant 2$ because for some processes the discontinuous exit prop-

* Institute of Mathematics, Wrocław University of Technology. Research supported by KBN Grant 2 P03 A 028 16.

erty depends on global geometric properties of ∂S (see [6] for the example). P. W. Millar gave a general solution of the exit problem in [6], but geometric conditions on S imposed there are complicated and rather restrictive.

The primary purpose of the present paper is to use the methods of [6] to formulate simple conditions on the local geometry of ∂S which yield the discontinuous exit property for a broad class of Lévy processes including rotation invariant processes with infinite Lévy measure and no Gaussian component. We have the following result (for the notation see the Preliminaries):

THEOREM 1. *Let $S \subset \boldsymbol{R}^d$ be a Lipschitz domain. Assume that $\{X_t\}_{t \geq 0}$ satisfies* **H** *and its resolvent is strong Feller. If*

$$\int_{V \cap B(0,1)} E^0 \tau_{B(0,|y|)} \nu(dy) = \infty$$

for every cone V with vertex 0, then

$$P^x(X_{\tau_S} \in \partial S) = 0 \quad \textit{for every } x \in S.$$

The hypotheses on $\{X_t\}_{t \geq 0}$ given in Theorem 1 are satisfied, e.g., by every rotation invariant process with the infinite Lévy measure and no Gaussian component. The same holds true for any transient stable process of type A having the scaling property under mild conditions on ν (see [6]).

For more general processes we relax the geometric condition on S imposed in [6] to cover such domains as tori; see Theorem 2 below. We also point out that conditions of Millar on S, if imposed at every point of ∂S, characterize Lipschitz domains in $\boldsymbol{R}^d$.

2. Preliminaries. For the rest of the paper, let $d \geq 2$. We denote by $\{X_t\}_{t \geq 0}$ the process with stationary independent increments in $\boldsymbol{R}^d$ and the characteristic function

$$E^0 \exp\{i(u, X_t)\} = \exp\{-t\Phi(u)\}, \quad u \in \boldsymbol{R}^d,\ t \geq 0,$$

where

$$\Phi(u) = i(a, u) + \frac{1}{2}(\sigma u, u) + \int_{\boldsymbol{R}^d} \left(1 - e^{i(u,y)} + \frac{i(u, y)}{1+|y|^2}\right) \nu(dy),$$

ν denotes the Lévy measure, $a \in \boldsymbol{R}^d$, and σ is a positive semidefinite symmetric matrix. As usual, E^x denotes the expectation with respect to the distribution P^x of the process starting from $x \in \boldsymbol{R}^d$. We always assume that sample paths of X_t are right-continuous and have left-hand limits a.s. The process is Markov with transition probabilities given by $P_t(x, A) = P^x(X_t \in A) = P^0(X_t \in A - x)$. It is well known that $\{X_t\}_{t \geq 0}$ is strong Markov with respect to the so-called "standard filtration", and quasi-left-continuous on $[0, \infty]$ (see, e.g., [1]).

For $x \in \boldsymbol{R}^d$ and $r > 0$, we let $B(x, r) = \{y \in \boldsymbol{R}^d\colon |x-y| < r\}$ and $\operatorname{dist}(x, A) = \inf\{|x-y|;\ y \in A\}$, $\operatorname{dist}(A, B) = \inf\{|x-y|;\ x \in A,\ y \in B\}$ for $A, B \subset \boldsymbol{R}^d$. As usual, A^c is the complement of A. For $A \subset \boldsymbol{R}^d$ we define $T(A) = \inf\{t > 0; X_t \in A\}$, the *first hitting time* of A, and $\tau_A = T(A^c)$, the *first exit time* of A. For $r > 0$ we put $\tau_r = \tau_{B(0,r)}$.

A cone in $\boldsymbol{R}^d$ with vertex at 0 is any set V that can be constructed as follows: take a closed ball B of radius $r > 0$ whose center is at the distance $r_0 > r$ from 0; then V consists of all the points that lie on the rays passing from 0 through B. The axis of V is the ray from 0 through the center of the ball, the vertex angle is $2\sin^{-1}(r/r_0)$. A cone with vertex at $x \in \boldsymbol{R}^d$ is a set of the form $x + V$, where V is a cone with vertex at 0. An open cone is the interior of a closed cone as just defined.

We say that a domain (a nonempty, connected and open set) S has the *outer cone property* if there exist a constant $H > 0$ and a cone V such that for every $Q \in \partial S$ there is a cone V_Q with vertex Q, isometric with V and satisfying $V_Q \cap B(Q, H) \subset S^c$.

A domain $S \subset \boldsymbol{R}^d$ is called a *Lipschitz domain* if it is bounded and for each Q in its boundary ∂S there are: a Lipschitz function Γ_Q: $\boldsymbol{R}^{d-1} \to \boldsymbol{R}$, an orthonormal coordinate system CS_Q, and a number $R_Q > 0$ such that if $z = (z_1, \ldots, z_d)$ in CS_Q coordinates, then

$$S \cap B(Q, R_Q) = \{z \colon z_d > \Gamma_Q(z_1, \ldots, z_{d-1})\} \cap B(Q, R_Q).$$

We note that by compactness of ∂S, the radii R_Q are not less than a constant $R_0 > 0$ (the *localization radius* of S), and the Lipschitz constants of the functions Γ_Q are not greater than a constant $\lambda < \infty$ (the *Lipschitz constant* of S).

From the Lipschitz condition it easily follows that a Lipschitz domain has the outer cone property.

DEFINITION 1. The process is said to satisfy *hypothesis* H if for each $\alpha \in (0, \pi)$ there are $h(\alpha) > 0$ and $r_0(\alpha) > 0$ such that for all cones V of vertex angle α and vertex at 0

$$E^0 \int_0^{\tau_r} \mathbf{1}_V(X_s)\, ds \geq h(\alpha) E^0 \tau_r \tag{1}$$

for all $r \in (0, r_0(\alpha))$.

Note that (1) is to hold for all cones with the same vertex angle, regardless of the direction of the cone. Any isotropic (rotation invariant) process has this property as well as processes with genuinely d-dimensional Gaussian component, stable processes of type A, and many others (see [6]).

3. Main results. Throughout this section, S denotes a domain in $\boldsymbol{R}^d$. We record some auxiliary results due to Millar. The proofs can be found in [6]. In what follows we assume that if $\{X_t\}_{t \geq 0}$ has no Gaussian component, then $\nu(\boldsymbol{R}^d) = \infty$.

If $a \in (0, 1)$, then there is a constant $K = K(a) > 0$ such that

$$E^0 \tau_{ar} \geqslant K(a) E^0 \tau_r \tag{2}$$

for all $r > 0$.

Furthermore, if the resolvent of the process $\{X_t\}_{t \geqslant 0}$ is strong Feller, then the function $f(x) = P^x(X_{\tau_S} \in A)$ is continuous on S for every Borel set $A \subset \boldsymbol{R}^d$

We call a point $y \in S$ *possible* if

$$P^x\big(T(B(y, \delta)) < \tau_S\big) > 0 \tag{3}$$

for all $x \in S$ and all $\delta > 0$. If $\{X_t\}_{t \geqslant 0}$ satisfy hypothesis H, then every point in S is possible.

We also introduce a *translation property*, which is a weakening of the translation property defined in [6]. The modification enables us to handle a more general class of domains.

Definition 2. Let V be a cone with vertex at 0 and $R > \varepsilon > 0$. We say that S has the *translation property relative to* V, R, ε if

(a) $0 \in S$,

(b) $\partial S \cap V \cap B(0, R) \subset B(0, R-\varepsilon)$,

(c) for every cone $V' \subset V$, $V' \neq V$, with the same axis,

$$[\partial S \cap V' \cap B(0, R)] - y \subset S$$

for all sufficiently small $y \in V$,

(d) there is a constant $c > 0$ such that for all sufficiently small $y \in V$ the distance between $[\partial S \cap V \cap B(0, R)] - y$ and $\partial S \cap V$ is at least $c|y|$.

The following result is a generalization of Theorem 3.1 in [6].

Theorem 2. *Let $S \subset \boldsymbol{R}^d$ be a domain with the outer cone property. Let V be an open cone such that S has the translation property relative to V, R, ε for some $R > \varepsilon > 0$. Assume that $\{X_t\}_{t \geqslant 0}$ satisfies hypothesis* H *and its resolvent is strong Feller. If $\int_{V \cap B(0,1)} E^0 \tau_{|y|} \nu(dy) = \infty$, then*

$$P^0\big(X_{\tau_S} \in \partial S \cap V \cap B(0, R)\big) = 0. \tag{4}$$

Proof. We generally follow the lines of the proof of Theorem 3.1 in [6], so we only give a sketch of the proof.

Assume that (4) does not hold, i.e. there is $\delta > 0$ such that

$$P^0\big(X_{\tau_S} \in \partial S \cap V \cap B(0, R)\big) = \delta.$$

Let $W = (\operatorname{int} S^c) \cap V \cap B(0, R)$. For $u, v \in \boldsymbol{R}^d$ we define $f(u, v) = \mathbf{1}_S(u) \mathbf{1}_W(v)$. We have

$$P^0(X_{\tau_S} \in W) = E^0 \int_0^{\tau_S} \int_{\boldsymbol{R}^d} f(X_s, X_s + y) \nu(dy) ds$$

(see, e.g., [4] or [6]).

Let V' be an open cone contained in V with the same axis and smaller vertex angle and such that $P^0(X_{\tau_S} \in \partial S \cap V' \cap B(0, R)) \geqslant \delta/2$. For fixed $y \in V$ we define

$$U_y = [\partial S \cap V' \cap B(0, R)] - y, \quad D_y = S \cap (W - y).$$

If $x \in \bar{U}_y = [\partial S \cap \bar{V}' \cap B(0, R)] - y$, then from the outer cone property and the translation property relative to V, R, ε we have $B(x, c|y|) \cap C_x \subset D_y$ for some cone C_x with vertex x and vertex angle $\beta \in (0, \pi)$ (β independent of x), provided y is sufficiently small, say $|y| < \eta$ for some η. Using this fact, the strong Markov property and hypothesis H we find that

$$E^0 \int_0^{\tau_S} f(X_s, X_s + y)\, ds \geqslant h(\beta) E^0 \tau_{c|y|} P^0(T(U_y) < \tau_S). \tag{5}$$

For $n \geqslant 1$ we let

$$\Delta_n = \{x \in S\colon \operatorname{dist}(x, \partial S) > 1/n\} \quad \text{and} \quad S_n = [S \cap V' \cap B(0, R - \varepsilon)] \cup \Delta_n.$$

By quasi-left-continuity we have $P^0(X_{\tau_{S_n}} \in \partial S \cap V' \cap B(0, R)) \geqslant \delta/4$ if n is sufficiently large. For such fixed n we have $S_n - y \subset S$ provided $\eta < 1/n$, which we may and do assume.

The function $y \mapsto P^y(X_{\tau_{S_n}} \in \partial S \cap V' \cap B(0, R))$ is continuous at 0, so, by taking smaller η we infer that, for $y \in V$ and $|y| < \eta$,

$$\begin{aligned} P^0(T(U_y) < \tau_S) &\geqslant P^0(T(U_y) = \tau_{(S_n - y)}, T(U_y) < \infty) \\ &\geqslant P^y(X_{\tau_{S_n}} \in \partial S \cap V' \cap B(0, R)) \geqslant \delta/8 \end{aligned} \tag{6}$$

because the resolvent of $\{X_t\}_{t \geqslant 0}$ is strong Feller.

Let $V_\eta = V \cap B(0, \eta)$. From (5), (6) and (2) we have

$$\begin{aligned} 1 \geqslant P^0(X_{\tau_S} \in W) &= E^0 \int_0^{\tau_S} \int_{\mathbf{R}^d} f(X_s, X_s + y)\, \nu(dy)\, ds \\ &\geqslant h(\beta) \frac{\delta}{8} \int_{V_\eta} E^0 \tau_{c|y|}\, \nu(dy) \geqslant h(\beta) \frac{\delta}{8} K(c) \int_{V_\eta} E^0 \tau_{|y|}\, \nu(dy) = \infty. \end{aligned}$$

This is the required contradiction. The proof of (4) is complete. ∎

We now digress on the consequences of the translation property, if imposed at every point of ∂S.

Definition 3. A domain $S \subset \mathbf{R}^d$ is called a *Millar domain* if it is bounded, has the outer cone property, and for each $Q \in \partial S$ there are: an open cone V_x with vertex $x \in S$ and numbers R, ε such that $R > \varepsilon > 0$, $Q \in V_x \cap B(x, R)$ and $S - x$ has the translation property relative to $V_x - x$, R, ε.

Lemma 1. *A set is a Millar domain if and only if it is a Lipschitz domain.*

Throughout the proof, for $z = (z_1, \ldots, z_{d-1}, z_d)$ we write $\tilde{z} = (z_1, \ldots, z_{d-1})$ and $\|\tilde{z}\|^2 = z_1^2 + \ldots + z_{d-1}^2$.

Proof. Let S be a Lipschitz domain with localization radius R_0 and Lipschitz constant λ. Choose $Q \in \partial S$. We may and do assume $Q = (0, \ldots, 0, -R_0/2)$. Let Γ_Q denote the Lipschitz function such that

$$S \cap B(Q, R_0) = \{z\colon z_d > \Gamma(\tilde{z})\} \cap B(Q, R_0).$$

Set $V = \{z\colon z_d < 0, \|\tilde{z}\| < \eta |z_d|\}$, where $\eta = \sqrt{\lambda^2+1}-\lambda$. The set V is an open cone with vertex 0. Let $R = R_0/\sqrt{\eta^2+1}+R_0/2$, $\varepsilon = R_0/2$.

We have $Q \in V \cap B(0, R)$. We will check that S has the translation property relative to V, R, ε.

(a) $0 \in S$.

(b) Let $w \in \partial S \cap V \cap B(0, R)$. Then $w = t\theta$, where $t \in (0, R)$ and $\theta \in V$, $|\theta| = 1$. We get

$$|w-Q|^2 = t^2+R_0^2/4+tR_0\,\theta_d < t^2+R_0^2/4-tR_0/\sqrt{\eta^2+1} < R_0^2,$$

so

$$\partial S \cap V \cap B(0, R) \subset B(Q, R_0), \tag{7}$$

and hence $w_d = \Gamma_Q(\tilde{w})$. We have $|w_d+R_0/2| = |\Gamma_Q(\tilde{w})-\Gamma_Q(0)| \leqslant \lambda \|\tilde{w}\|$ and $\|\tilde{w}\| < -\eta w_d$. From this we conclude that

$$\|\tilde{w}\| < \frac{R_0/2}{(1/\eta)-\lambda}$$

and $|w|^2 = \|\tilde{w}\|^2+w_d^2 < R_0^2/(1+\eta^2) = (R-\varepsilon)^2$, so

$$\partial S \cap V \cap B(0, R) \subset B(0, R-\varepsilon).$$

(c) Fix $y \in V$, $|y| < R_0/2$ and $w \in \partial S \cap V \cap B(0, R)$. The same method as for (7) shows that $\partial S \cap V \cap B(0, R-\varepsilon) \subset B(Q, R_0/2)$, and this yields $w-y \in B(Q, R_0)$. We have $|\Gamma_Q(\tilde{w})-\Gamma_Q(\tilde{w}-\tilde{y})| \leqslant \lambda \|\tilde{y}\|$, so $\Gamma_Q(\tilde{w}-\tilde{y}) < w_d+\lambda\eta |y_d| < w_d-y_d$. Hence $w-y \in S$.

(d) Let $y \in V$, $|y| < \varepsilon/2$ and $w \in \partial S \cap V \cap B(0, R)$. We have $w-y \in B(0, R-\varepsilon/2)$, so

$$\operatorname{dist}\big(w-y,\ \partial S \cap V \cap B^c(0, R)\big) \geqslant \varepsilon/2 > |y|.$$

Let $Z_w = \{z\colon |z_d-w_d| \leqslant \lambda \|\tilde{z}-\tilde{w}\|\}$. We have $\partial S \cap V \cap B(0, R) \subset Z_w$ and $w-y \notin Z_w$. Hence

$$\operatorname{dist}\big(w-y,\ \partial S \cap V \cap B(0, R)\big) \geqslant \operatorname{dist}(w-y,\ \partial Z_w).$$

An easy computation shows that

$$\operatorname{dist}(w-y,\ \partial Z_w) > \frac{1-\lambda\eta}{\sqrt{(\lambda^2+1)(\eta^2+1)}}|y|,$$

so we may put

$$c = \frac{1-\lambda\eta}{\sqrt{(\lambda^2+1)(\eta^2+1)}} \wedge 1.$$

The first part of the proof is complete.

Now, let S denote a Millar domain.

Choose $Q \in \partial S$ and let a cone V_x with vertex x and R, ε, $R > \varepsilon > 0$, be such that $Q \in V_x \cap B(x, R)$ and $S-x$ has the translation property relative to $V_x - x$, R, ε. We may and do assume that $x = 0$, $Q = (0, \ldots, 0, q_d)$; $q_d < 0$ and $V_x = \{z\colon z_d < 0,\ \|\tilde{z}\| < \eta_x |z_d|\}$, $\eta_x > 0$.

Set $V = \{z\colon z_d < 0,\ \|\tilde{z}\| < \eta |z_d|\}$, where $0 < \eta < \eta_x$. Let $r > 0$ be such that $[\partial S \cap V \cap B(0, R)] - y \subset S$ for each $y \in V_x$, $|y| < r$. Set

$$R_1 = r/2 \wedge \operatorname{dist}(Q, V^c) \wedge \operatorname{dist}\big(Q, B^c(0, R)\big)$$

and for $w \in \partial S \cap B(Q, R_1)$ let

$$M_w = (-V+w) \cap B(Q, R_1), \qquad N_w = (V+w) \cap B(Q, R_1).$$

Using the outer cone property and (c) from the definition of the translation property we get

$$\text{(8)} \qquad M_w \subset S \quad \text{and} \quad N_w \subset \operatorname{int} S^c$$

for each $w \in \partial S \cap B(Q, R_1)$. Set $R_Q = R_1 \eta/\sqrt{\eta^2+1}$. For every $z \in B(Q, R_Q)$ there exists $z^* \in \boldsymbol{R}$ such that $(\tilde{z}, z^*) \in \partial S \cap B(Q, R_1)$. Moreover, from (8) it follows that there is only one such z^*. Therefore, we can define a function $\Gamma_Q(\tilde{z}) = z^*$, $z \in B(Q, R_Q)$. Using (8) again we infer that Γ_Q is the Lipschitz function with Lipschitz constant $\lambda = 1/\eta$ and

$$S \cap B(Q, R_Q) = \{z\colon z_d > \Gamma_Q(\tilde{z})\} \cap B(Q, R_Q). \ \blacksquare$$

Lemma 1 is a motivation to consider Lipschitz domains in the context of our exit problem. In a sense, Lipschitz domains are the most general domains for which Millar's methods may be used to prove the discontinuous exit property for typical Lévy processes.

LEMMA 2. *Let $A \subset \boldsymbol{R}^d$ be a Borel set. If $\{X_t\}_{t \geqslant 0}$ satisfies hypothesis* **H**, *its resolvent is strong Feller, and there exists $x \in S$ such that $P^x(X_{\tau_S} \in A) = 0$, then $P^z(X_{\tau_S} \in A) = 0$ for all $z \in S$.*

Proof. By the strong Markov property, for each $z \in S$ and $\delta > 0$ we have

$$\begin{aligned} 0 = P^x(X_{\tau_S} \in A) &\geqslant P^x\big(X_{\tau_S} \in A;\ T\big(\overline{B(z, \delta)}\big) < \tau_S\big) \\ &= E^x\big(P^{X_{T(\overline{B(z,\delta)})}}(X_{\tau_S} \in A);\ T\big(\overline{B(z, \delta)}\big) < \tau_S\big). \end{aligned}$$

From (3) we get $P^x\big(T\big(\overline{B(z, \delta)}\big) < \tau_S\big) > 0$, so there exists $w \in \overline{B(z, \delta)}$ such that $P^w(X_{\tau_S} \in A) = 0$. This yields that the set $\{w \in S\colon P^w(X_{\tau_S} \in A) = 0\}$ is dense in S.

From continuity of the function $z \to P^z(X_{\tau_S} \in A)$ we infer that $P^z(X_{\tau_S} \in A) = 0$ for each $z \in S$. ∎

Proof of Theorem 1. Let $Q \in \partial S$. By Lemma 1 there exists $x \in S$, an open cone V_x with vertex x and numbers R, ε such that $Q \in V_x \cap B(x, R)$ and $S - x$ has the translation property relative to $V_x - x$, R, ε. We have

$$\int_{(V_x - x) \cap B(0,1)} E^0 \tau_{|y|} \nu(dy) = \infty.$$

By Theorem 2 we get

$$P^0\big(X_{\tau_{(S-x)}} \in \partial(S-x) \cap (V_x - x) \cap B(0, R)\big) = 0$$

or

$$P^x\big(X_{\tau_S} \in \partial S \cap V_x \cap B(x, R)\big) = 0.$$

Lemma 2 yields

$$P^z\big(X_{\tau_S} \in \partial S \cap V_x \cap B(x, R)\big) = 0 \quad \text{for every } z \in S.$$

By the usual compactness argument, Theorem 1 holds true. ∎

Acknowledgments. The author is grateful to Prof. T. Byczkowski and Dr. K. Bogdan for introducing him into subject and many helpful suggestions during the preparation of the paper.

REFERENCES

[1] R. M. Blumenthal and R. K. Getoor, *Markov Processes and Potential Theory*, Pure Appl. Math., Academic Press Inc., New York 1968.

[2] K. Bogdan, *The boundary Harnack principle for the fractional Laplacian*, Studia Math. 123 (1997), pp. 43–80.

[3] N. Ikeda and S. Watanabe, *On some relations between the harmonic measure and the Lévy measure for a certain class of Markov processes*, J. Math. Kyoto Univ. 2-1 (1962), pp. 79–95.

[4] N. Ikeda and S. Watanabe, *Stochastic differential equations and diffusion processes*, North-Holland Publishing Company, Amsterdam–Oxford–New York 1981.

[5] P. W. Millar, *Exit properties of stochastic processes with stationary independent increments*, Trans. Amer. Math. Soc. 178 (1973), pp. 459–479.

[6] P. W. Millar, *First passage distributions of processes with independent increments*, Ann. Probab. 3, No. 2 (1975), pp. 215–233.

Institute of Mathematics
Wrocław University of Technology
Wybrzeże Wyspiańskiego 27
50-370 Wrocław, Poland
E-mail: sztonyk@im.pwr.wroc.pl

Received on 14.2.2000;
revised version on 9.6.2000